Redesigning Electroweak Theory: Does the Higgs Particle Exist?

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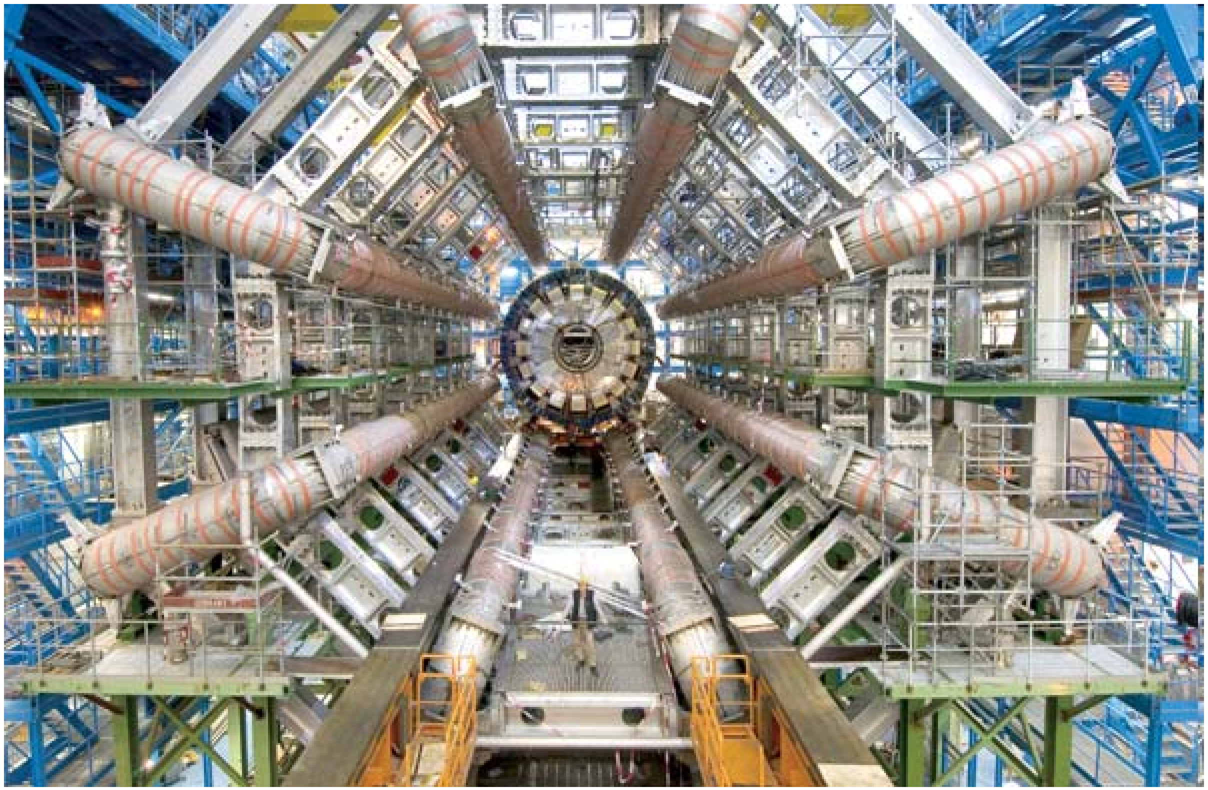
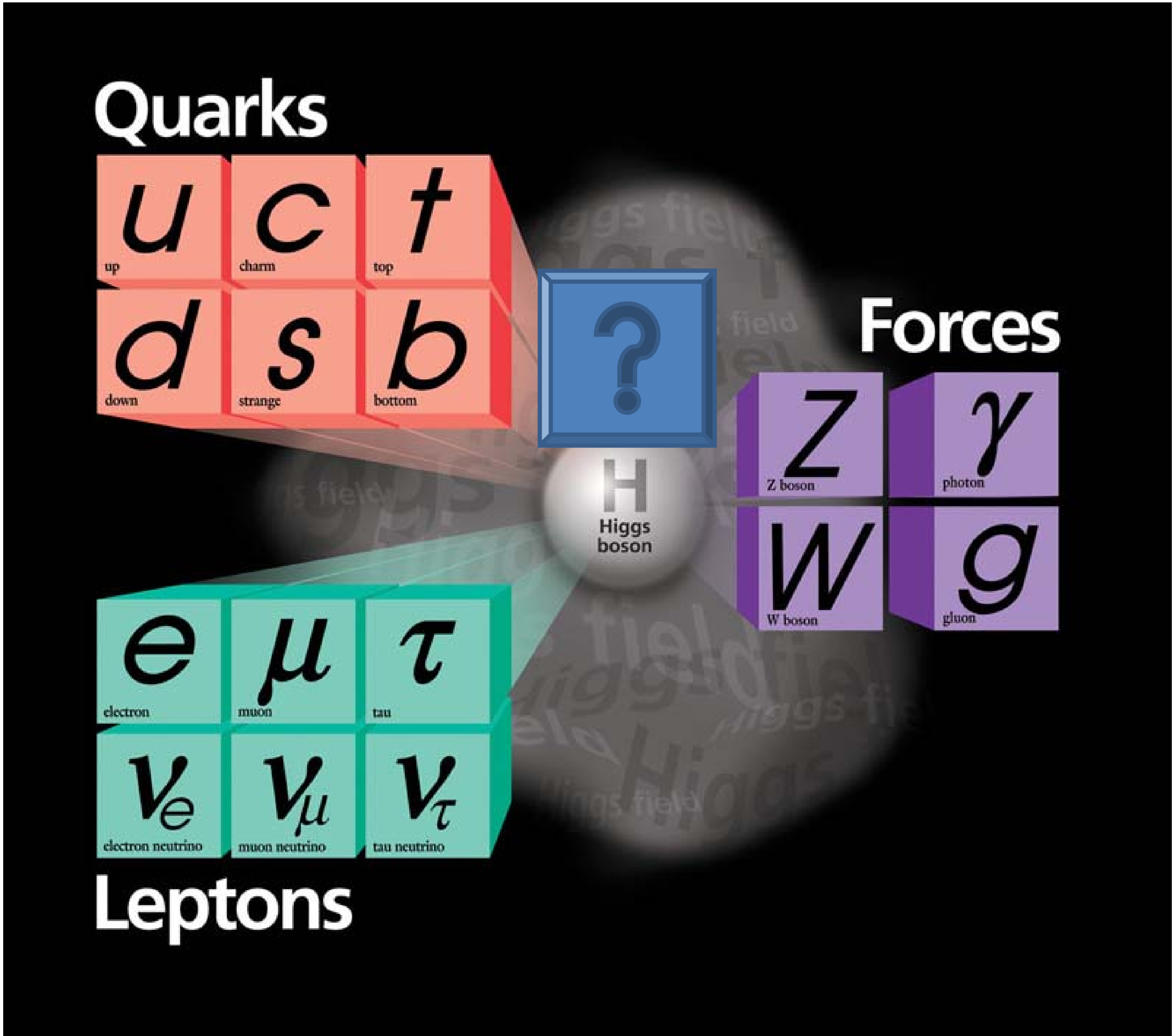
The ATLAS experiment at CERN. (Courtesy CERN)



1. Standard Electroweak Model With A Higgs Particle

- The origin of the symmetry breaking mechanism in electroweak theory remains elusive after almost 50 years. The standard and commonly accepted explanation is that the symmetry $SU_L(2) \times U_Y(1)$ is not broken by the interactions but is "softly", spontaneously broken by the asymmetry of the ground state (vacuum state).

- The standard electroweak (EW) model gains mass for the W and Z bosons, while keeping the photon massless by introducing a classical scalar field into the action. This scalar degree of freedom is assumed to transform as an isospin doublet, spontaneously breaking the $SU_L(2) \times U_Y(1)$ by a Higgs mechanism at the purely classical tree graph level.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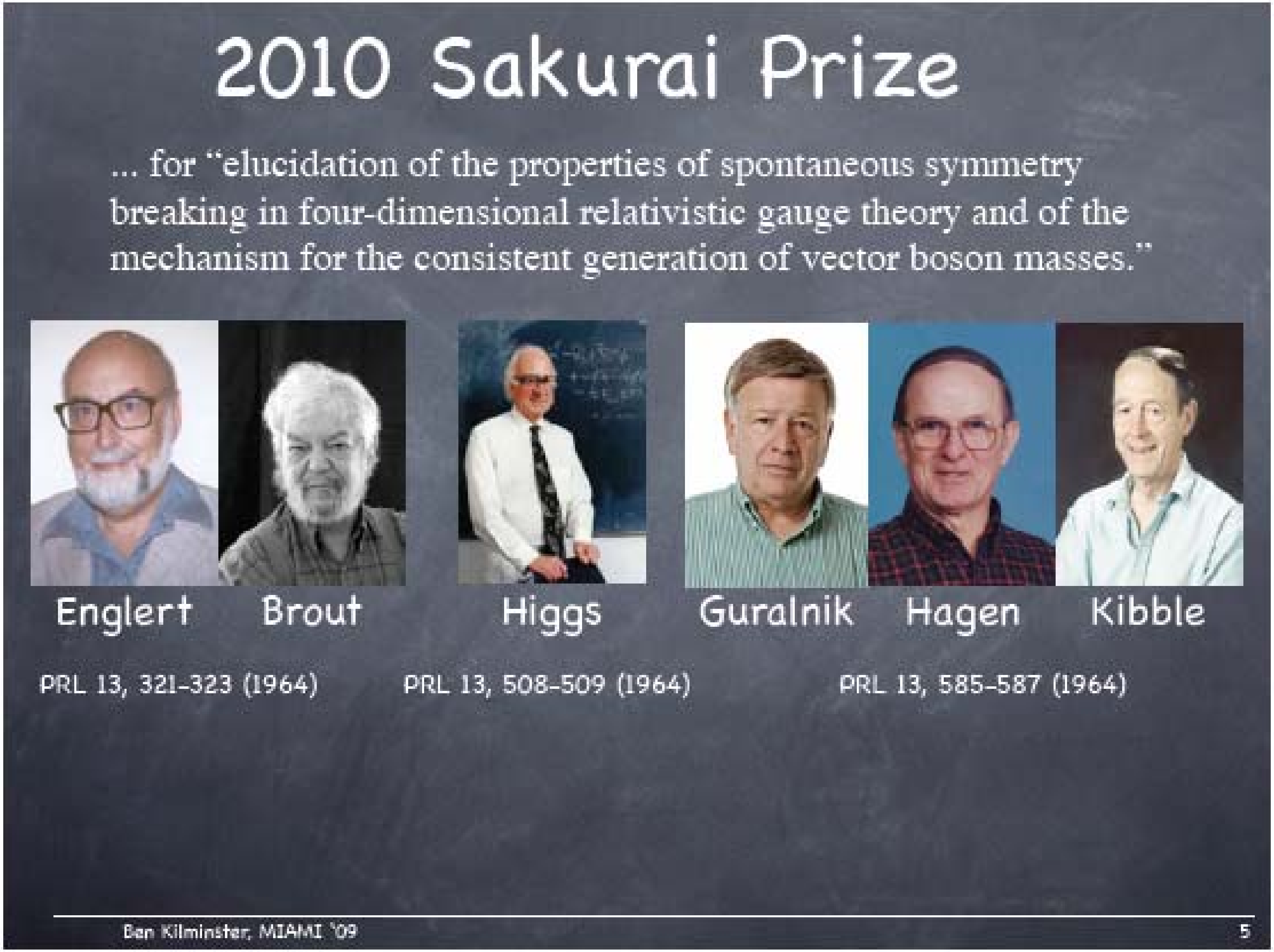

2010 Sakurai Prize
... for "elucidation of the properties of spontaneous symmetry breaking in four-dimensional relativistic gauge theory and of the mechanism for the consistent generation of vector boson masses."
Englert
Brout
Higgs
Guralnik
Hagen
Kibble
PRL 13, 321-323 (1964)
PRL 13, 508-509 (1964)
PRL 13, 585-587 (1964)
Ben Kilminster, MIAMI '09
5

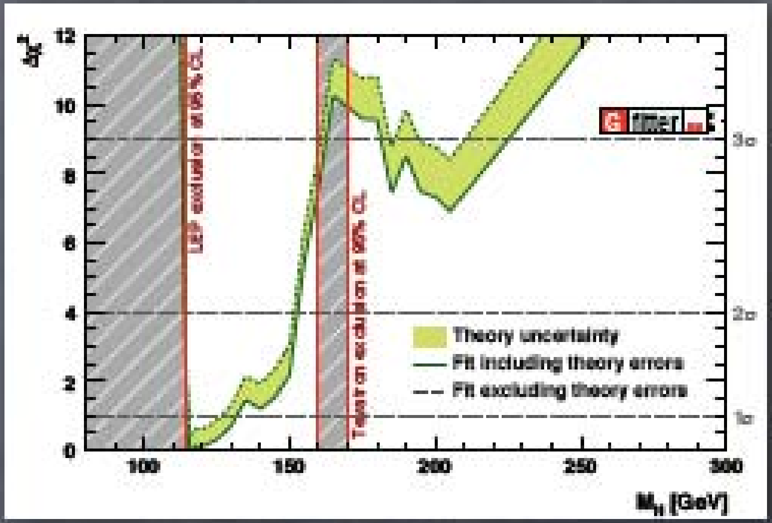
- The minimal EW model with a Higgs doublet is consistent with the experimental bounds on flavor changing neutral currents and CP violation: $Br = \Gamma(b \to s\gamma)/\Gamma(b \to Xe\nu) = (3.55_{-0.46}^{+0.53}) \times 10^{-3}$.

- The primary target of an EW global fit is the prediction of the Higgs mass. The *complete fit* represents the most accurate estimation of M_H considering all available data (arXiv:0811.0009[CERN]). The result is $M_H = 116.4_{-1.3}^{+18.3}$ GeV where the error accounts for both experimental and theoretical uncertainties. The result for the *standard fit* without the direct Higgs searches is $M_H = 80_{-23}^{+30}$ GeV and the 2σ and 3σ intervals are, respectively, [39, 155] GeV and [26, 209] GeV.

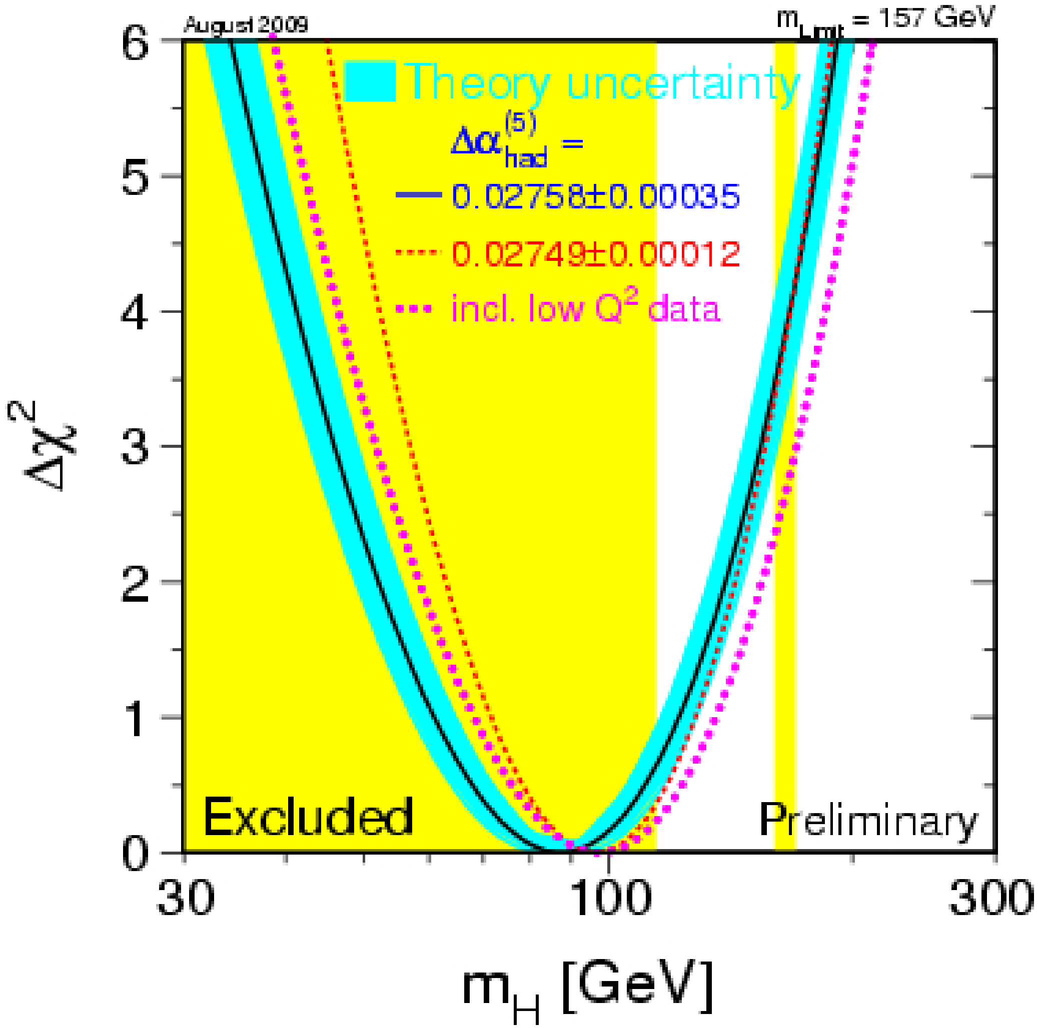
- We conclude from this that the minimal EW model requires a light Higgs.

All Higgs constraints together

- Gfitter group includes
  - Indirect electroweak constraints (M_t, M_W + Z, W properties, left-right asymmetry, $\sin^2 \theta_W$)
  - LEP direct Higgs searches
  - Tevatron Higgs searches
    - Likelihood ratio from searches
    - Not including results shown today



Higgs excluded region



- For a classical potential for a scalar field φ we can identify: $<T_{\mu\nu}>_0 = V(\varphi) = \rho_{vac}$. The Higgs field vacuum energy is calculated from the classical Higgs potential:

$$V(\phi) = V_0 - \mu^2\phi^2 + \lambda\phi^4 \qquad \langle 0|\phi|0\rangle = v$$

where v ~ 250 GeV is the EW energy scale. We have $\mu^4 = \lambda^2 v^4$. From the minimization of the Higgs potential we obtain $\varphi^2_{min} = \mu^2/2\lambda$ and $V_{min} = V_0 - \mu^4/4\lambda = \rho_{vac}^{ssb}$. Choosing V(0) = 0 we obtain

$$\rho_{\rm vac}^{\rm SSB} = -\frac{\mu^4}{4\lambda} \sim -\lambda v^4 \sim -10^5\,{\rm GeV}^4. \ \rho_{\rm vac}^{\rm OBS} \sim 10^{-47}\,GeV^4. \ |\rho_{\rm vac}^{\rm SSB}| \sim 10^{56}\rho_{\rm vac}^{\rm OBS}.$$

(Vacuum energy) = (Vacuum zero-point energy)+ (The Higgs potential) +(QCD gluon and quark condensates)

Problems with the Higgs particle in the standard model:

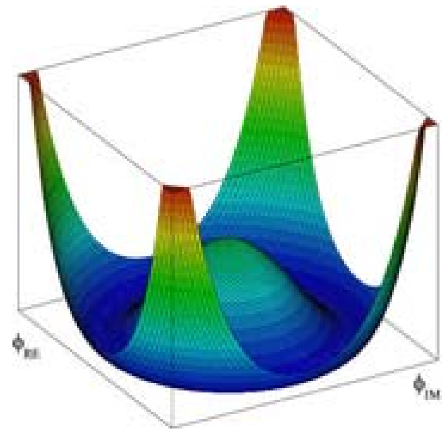
$$V_{\text{Higgs}} = V_0 - \mu^2\phi^2 + \lambda\phi^4 + (\bar{\psi}_{Li} Y_{ij} \psi_{Rj} \phi + h.c.)$$

Vacuum energy – V_0=0 then $\rho_{vac\ ssb} \sim 10^{56}$ too large vacuum energy compared to observation

Scalar field quadratic divergence – hierarchy problem

Possible instability problems depending on the size of M_H

The flavor problem: large unexplained ratios of Y_{ij} Yukawa coupling constants



• Discovering a satisfactory alternative has proved to be highly non-trivial. Proposed alternatives face severe problems. New particle contributions at less than 1 or 2 TeV level can affect precision EW data that can generate unacceptably large effects; significant fine-tuning may be required at least at the 1-percent level. These models include MSSM, Little Higgs, pseudo-Goldstone bosons, higher-dimensional models. Extensions of the standard EW model such as technicolor, and other composite models can face unacceptably large flavor changing contributions and CP violation.

2. Gauge Invariant Local Massless EW Theory

• We shall use the metric convention, $\eta_{\mu\nu}$= diag(+1,-1,-1,-1), and set ħ= c = 1. The theory is based on the local $SU_L(2) \times U_Y(1)$ invariant Lagrangian that includes leptons and quarks (with the color degree of freedom of the strong interaction group $SU_c(3)$) and the boson vector fields that arise from gauging the global $SU_L(2) \times U_Y(1)$ symmetries:

$$L_{\text{local}} = L_F + L_W + L_B + L_I \, .$$

• L_F is the free fermion Lagrangian consisting of massless kinetic terms for each fermion:

$$L_F = \sum_{\psi} \bar{\psi} i \partial\!\!\!/ \psi = \sum_{q^L} \bar{q}^L i \partial\!\!\!/ q^L + \sum_{f} \bar{\psi}^R i \partial\!\!\!/ \psi^R,$$

$$q^L \in \left[\begin{pmatrix} \nu^L \\ e^L \end{pmatrix}, \begin{pmatrix} u^L \\ d^L \end{pmatrix}_{r,g,b} \right] \qquad L_B = -\frac{1}{4} B^{\mu\nu} B_{\mu\nu} \qquad B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$$

$$L_W = -\frac{1}{4} W^a_{\mu\nu} W^{a\mu\nu}, \qquad W^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu - g f^{abc} W^b_\mu W^c_\nu$$

- Note that there is no classical scalar field contribution in the Lagrangian.
- The SU(2) generators satisfy

$$[T^a, T^b] = i f^{abc} T^c, \qquad \text{with} \qquad T^a = \frac{1}{2}\sigma^a$$

- The fermion gauge-boson interaction terms are

$$L_I = -g J^{a\mu} W^a_\mu - g' J^\mu_Y B_\mu \qquad J^{a\mu} = \sum_{q^L} \bar{q}^L \gamma^\mu T^a q^L, \qquad \text{and} \qquad J^\mu_Y = \sum_\psi \frac{1}{2} Y_\psi \bar{\psi} \gamma^\mu \psi$$

$$Y(q^L_{\text{lepton}}) = -1, \quad Y(q^L_{\text{quark}}) = \frac{1}{3}, \quad Y(e^R) = -2, \quad Y(\nu^R) = 0, \quad Y(u^R) = \frac{4}{3}, \quad Y(d^R) = \frac{2}{3}.$$

- L is invariant under the local gauge transformations (order g, g'):

$$\delta\psi^L = -\left(igT^a\theta^a + ig'\frac{Y_\psi}{2}\beta\right)\psi^L, \qquad \delta\psi^R = -ig'\frac{Y_\psi}{2}\beta\psi^R,$$

$$\delta W^a_\mu = \partial_\mu\theta^a + gf^{abc}\theta^b W^c_\mu, \qquad \delta B_\mu = \partial_\mu\beta$$

- L is an $SU(2)_L \times U(1)_Y$ invariant Lagrangian.
- Quantization is accomplished via the path integral formalism:

$$\langle T(\mathcal{O}[\phi])\rangle \propto \int [D\bar{\psi}][D\psi][DW][DB]\mu_{\text{inv}}[\bar{\psi}, \psi, B, W]\mathcal{O}[\phi]\exp\left(i\int d^4x L_{\text{local}}\right)$$

- In the local case the invariant measure μ_{inv} is the trivial one.

- We have to gauge fix the Lagrangian:

$$L_{\rm GF} = -\frac{1}{2\xi}(\partial_\mu B^\mu)^2 - \frac{1}{2\xi}(\partial_\mu W^{a\mu})^2.$$

- We look at diagonalizing the charged sector and mixing in the neutral boson sector. If we write

$$W^\pm = \frac{1}{\sqrt{2}}(W^1 \mp iW^2)$$

then we get the fermion interaction terms: $-\frac{g}{\sqrt{2}}(J^+_\mu W^{+\mu} + J^-_\mu W^{-\mu}),$

$$J^\pm_\mu = J^\pm_{1\mu} \pm iJ^\pm_{2\mu} = \sum_{q_L} \bar{q}^L \gamma_\mu T^\pm q^L \qquad J^+_\mu = \sum_{q_L} (\bar{\nu}^L \gamma_\mu e^L + \bar{u}^L \gamma_\mu d^L).$$

- In the neutral sector, we can mix the fields in the usual way:

$$Z_\mu = c_w W_\mu^3 - s_w B_\mu \quad \text{and} \quad A_\mu = c_w B_\mu + s_w W_\mu^3, \quad s_w = \sin\theta_w \quad c_w = \cos\theta_w$$

$$s_w^2 = \frac{g'^2}{g^2 + g'^2} \quad \text{and} \quad c_w^2 = \frac{g^2}{g^2 + g'^2}.$$

- The neutral current fermion interaction terms now look like:

$$-gJ^{3\mu}W_\mu^3 - g'J_Y^\mu B_\mu = -(gs_w J^{3\mu} + g'c_w J_Y^\mu)A_\mu - (gc_w J^{3\mu} - g's_w J_Y^\mu)Z_\mu.$$

- We have the unification condition: $e = gs_w = g'c_w$

$$J_{\text{em}}^\mu = J^{3\mu} + J_Y^\mu, \qquad J_{\text{NC}}^\mu = J^{3\mu} - s_w J_{\text{em}}^\mu,$$

$$L_I = -\frac{g}{\sqrt{2}}(J_\mu^+ W^{+\mu} + J_\mu^- W^{-\mu}) - gs_w J_{\text{em}}^\mu A_\mu - \frac{g}{c_w} J_{\text{NC}}^\mu Z_\mu$$

3. Gauge Invariant Regularized Massless Theory

• To regularize the fields, we write the non-local (smeared) fields as a convolution of the local fields with a function whose Euclidean momentum space Fourier transform is an entire function. This function can be related to a Lorentz invariant operator distribution as

$$\Phi(x) = \int d^4y G(x-y)\phi(y) = G\left(\frac{\Box}{\Lambda_W^2}\right)\phi(x), \qquad G\left(\frac{\Box}{\Lambda_W^2}\right) \equiv \mathcal{E}_m = \exp\left(-\frac{\Box + m^2}{2\Lambda_W^2}\right).$$

• We now write the initial Lagrangian in non-local form:

$$L_{\rm reg} = L[\phi]_F + \mathcal{L}[\Phi]_I,$$

where $\mathcal{L}[\Phi]_I$ indicates smearing of the interacting fields.

• An essential feature of the regularized, non-local field theory is the requirement that the classical tree graph theory remain local, giving us a well defined classical limit in the gauge invariant case. JWM, Phys. Rev. D 41, 1177 (1990); D. Evens, JWM, G. Kleppe, and R. P. Woodard, Phys. Rev. D 43, 499 (1991); JWM, Mod. Phys. Lett. A6, 1011 (1991); M. Clayton and JWM, Mod. Phys. Lett. A6, 2697 (1991); JWM, ArXiv 0709.4269 [hep-ph] (2007); JWM and V. T. Toth, ArXiv 0812.1991 [hep-ph] (2008); JWM and V. T. Toth, ArXiv 0812.1994 [hep-ph] (2008).

• We first note that we must alter the quantized form of the theory by generalizing the path integral (including gauge fixing ghost fields):

$$\langle T^*(\mathcal{O}[\Phi])\rangle \propto \int [D[\phi]\mu_{\mathrm{meas}}[\phi,\Phi]\mathcal{O}[\Phi]\exp(iS_0[\phi]+iS_I[\Phi])$$

$$W[\mathcal{J}] = \ln[Z[\mathcal{J}]) = \ln(\int[\phi]\mu_{\mathrm{meas}}[\phi,\Phi]\exp(i\int dx\{L_F[\phi]+\mathcal{L}[\Phi]+\mathcal{J}\Phi\}))$$

• We note that in momentum space, the smeared fields are related one-to-one to the local fields:

$$\Phi(p) = G(p^2)\phi(p) = \exp\left(\frac{p^2-m^2}{2\Lambda_W^2}\right)\phi(p),$$

• The nonlocal regularization can produce gauge invariant, Poincar\’e invariant, unitary and finite loop graphs to all orders of perturbation theory for QED (D. Evens, JWM, G. Kleppe, R. P. Woodard, Phys. Rev. D 43, 499 (1991)). The EW theory can satisfy BRST gauge invariance, Poincar\’e invariance and unitarity and produce finite loop diagrams for the massless SU(2) X U(1), although a direct proof of gauge invariance and BRST invariance for the quantum measure is achieved to order g^2 (For Yang-Mills see: G. Kleppe and R. P. Woodard, Nucl. Phys. B388, 81 (1992)).

• If δ_ε represents a gauge transformation by ε (we are only worried about the *invariance of the functional measure* here), then

$$\int (D[\phi])\delta_\epsilon(\mu[\phi]\exp(iS)) = 0$$

If the gauge transformation corresponds to an actual gauge symmetry:

$$\int \delta_\epsilon(\mu[\phi]\exp(i(S + S_{\rm gf})) = 0$$

4. Breaking The Symmetry With A Path Integral Measure

- We break $SU_L(2) \times U_Y(1)$ down to $U_{em}(1)$ not at the classical level as is done in the standard model, which generates boson masses at tree level, but in the quantum regime, so that all the effects show up at loop order (which is where the non-locality shows up as well, as both are quantum effects) (JWM, Mod. Phys. Lett. A6, 1011 (1991).) This means leaving the action gauge invariant and modifying the measure, which alters the quantization of the theory, in order to produce the desired results. For δ_ε which represents a gauge transformation by ε and a measure μ[φ]:

$$\int D[\phi]\delta_\epsilon(\mu[\phi]\exp(iS)) = \int d^4x\epsilon\mathcal{F}[\phi]\mu[\phi]\exp(iS)$$

where $\mathcal{F}[\phi]$ is a functional of the fields [φ], we have a Ward-Takahashi identity for the broken symmetry SU(2) × U(1).

- The quantum symmetry breaking measure in our path integral generates three new degrees of freedom as Nambu-Goldstone modes that give the $W^\pm$ and Z^0 bosons longitudinal modes, which makes them massive while retaining a massless photon.

• Since we want to mix the W_3 and B to get a massive Z and a photon, we need to work with the measure in a sector which is common to all gauge bosons. This implies working with the fermion contributions and leaving the bosonic and ghost contributions invariant.

• The self-energy contribution coming from

is given by

$$
\begin{aligned}
-i\Pi_f^L &= -\frac{4iee'\Lambda_W^2}{(4\pi)^2}[g_+(K_{m_1m_2} - L_{m_1m_2}) + g_- M_{m_1m_2}], \\
-\Pi_f^T &= -\frac{4iee'\Lambda_W^2}{(4\pi)^2}[g_+(K_{m_1m_2} - L_{m_1m_2} + 2P_{m_1m_2}) + g_- M_{m_1m_2}],
\end{aligned}
$$

where we define

$$
\begin{aligned}
K_{m_1 m_2} &= \int_0^{\frac{1}{2}} d\tau (1-\tau) \left[\exp\left(-\tau \frac{p_E^2}{\Lambda_W^2} - f_{m_1 m_2} \right) + \exp\left(-\tau \frac{p_E^2}{\Lambda_W^2} - f_{m_2 m_1} \right) \right], \\
P_{m_1 m_2} &= -\frac{p_E^2}{\Lambda_W^2} \int_0^{\frac{1}{2}} d\tau \tau (1-\tau) \left[E_1 \left(\tau \frac{p_E^2}{\Lambda_W^2} + f_{m_1 m_2} \right) + E_1 \left(\tau \frac{p_E^2}{\Lambda_W^2} + f_{m_2 m_1} \right) \right], \\
L_{m_1 m_2} &= \int_0^{\frac{1}{2}} d\tau (1-\tau) \left[f_{m_1 m_2} E_1 \left(\tau \frac{p_E^2}{\Lambda_W^2} + f_{m_1 m_2} \right) + f_{m_2 m_1} E_1 \left(\tau \frac{p_E^2}{\Lambda_W^2} + f_{m_2 m_1} \right) \right], \\
M_{m_1 m_2} &= \frac{m_1 m_2}{\Lambda_W^2} \int_0^{\frac{1}{2}} d\tau \left[E_1 \left(\tau \frac{p_E^2}{\Lambda_W^2} + f_{m_1 m_2} \right) + E_1 \left(\tau \frac{p_E^2}{\Lambda_W^2} + f_{m_2 m_1} \right) \right],
\end{aligned}
$$

$$
f_{m_1 m_2} = \frac{m_1^2}{\Lambda_W^2} + \frac{\tau}{1-\tau} \frac{m_2^2}{\Lambda_W^2}.
$$

- If we insert this into the quadratic terms and invert, we get the corrected propagators (in a general gauge):

$$
iD^{\mu\nu} = -i \left(\frac{\eta^{\mu\nu} - \frac{p^\mu p^\nu}{p^2}}{p^2 - \Pi_f^T} + \frac{\frac{\xi p^\mu p^\nu}{p^2}}{p^2 - \xi \Pi_f^L} \right)
$$

• When the longitudinal piece Π_L is nonzero in the unitary gauge (where only the physical particle spectrum remains), we have no unphysical poles in the longitudinal sector. In this way, we can assure ourselves that we are not introducing spurious degrees of freedom into the theory.

• In the diagonalized $W^{\pm}$ sector, we get

$$-i\Pi^{L}_{W^{\pm}f} = -\frac{ig^2\Lambda_W^2}{(4\pi)^2}\sum_{q^L}(K_{m_1m_2} - L_{m_1m_2}),$$
$$-i\Pi^{T}_{W^{\pm}f} = -\frac{ig^2\Lambda_W^2}{(4\pi)^2}\sum_{q^L}(K_{m_1m_2} - L_{m_1m_2} + 2P_{m_1m_2}).$$

• We can now postulate a measure for the fermionic contribution to the W boson self-energies that is not gauge invariant. This introduces three Nambu-Goldstone degrees of freedom into the W sector and the W bosons acquire a longitudinal part and a corresponding mass. To see this, note that

$$-i\Pi^{L}_{W^{\pm}f}\Big|_{p^2=0} = -i\Pi^{T}_{W^{\pm}f}\Big|_{p^2=0} = -\frac{ig^2\Lambda_W^2}{(4\pi)^2}\sum_{q^L}(K_{m_1m_2} - L_{m_1m_2})\Big|_{p^2=0} \neq 0.$$

- In the B sector we have

$$
\begin{aligned}
-i\Pi^L_{Bf} &= -\frac{1}{2}\frac{ig'^2\Lambda_W^2}{(4\pi)^2}\sum_\psi[16(Q-T^3)^2(K_{mm}-L_{mm})+32Q(Q-T^3)M_{mm}],\\
-i\Pi^T_{Bf} &= -\frac{1}{2}\frac{ig'^2\Lambda_W^2}{(4\pi)^2}\sum_\psi[16(Q-T^3)^2(K_{mm}-L_{mm}+2P_{mm})+32Q(Q-T^3)M_{mm}],
\end{aligned}
$$

- We write the measure contribution as

$$
\Upsilon^{BB}_{\mu\nu} = -\frac{ig'^2\Lambda_W^2}{(4\pi)^2}\eta_{\mu\nu}\sum_\psi\left[\left(\frac{1}{2}-8(Q-T^3)^2\right)(K_{mm}-L_{mm})-16Q(Q-T^3)M_{mm}\right]
$$

Then we have

$$
\begin{aligned}
-i\Pi^L_{Bf} &= -\frac{1}{2}\frac{ig'^2\Lambda_W^2}{(4\pi)^2}\sum_\psi(K_{mm}-L_{mm}),\\
-i\Pi^T_{Bf} &= -\frac{1}{2}\frac{ig'^2\Lambda_W^2}{(4\pi)^2}\sum_\psi[(K_{mm}-L_{mm})+32(Q-T^3)^2P_{mm}].
\end{aligned}
$$

- The B - W_3 mixing sector originally looks like

$$-i\Pi^{L}_{W^3Bf} = -\frac{4igg'\Lambda_W^2}{(4\pi)^2}\sum_{\psi}[T^3(Q-T^3)(K_{mm}-L_{mm})+QM_{mm}],$$

$$-i\Pi^{T}_{W^3Bf} = -\frac{4igg'\Lambda_W^2}{(4\pi)^2}\sum_{\psi}[T^3(Q-T^3)(K_{mm}-L_{mm}+2P_{mm})+QM_{mm}].$$

- Thus, to make the mass contributions look identical, we find for the measure

$$\Upsilon^{W^3B}_{\mu\nu} = -\frac{igg'\Lambda_W^2}{(4\pi)^2}\eta_{\mu\nu}\sum_{\psi}\left[\left(-\frac{1}{2}-4T^3(Q-T^3)\right)(K_{mm}-L_{mm})-4QM_{mm}\right].$$

- Then we have

$$-i\Pi^{L}_{W^3Bf} = \frac{1}{2}\frac{igg'\Lambda_W^2}{(4\pi)^2}\sum_{\psi}(K_{mm}-L_{mm}),$$

$$-i\Pi^{T}_{W^3Bf} = \frac{1}{2}\frac{igg'\Lambda_W^2}{(4\pi)^2}\sum_{\psi}[(K_{mm}-L_{mm})-8T^3(Q-T^3)P_{mm}].$$

- Only the diagonal Z - Z piece has a longitudinal part

$$-i\Pi^L_{Zf} = -\frac{1}{2}\frac{i(g^2+g'^2)\Lambda_W^2}{(4\pi)^2}\sum_\psi (K_{mm}-L_{mm}). \qquad Z_\mu = c_w W^3_\mu - s_w B_\mu \quad \text{and} \quad A_\mu = c_w B_\mu + s_w W^3_\mu$$

- For the Z - Z part we get

$$-i\Pi^T_{Zf} = -\frac{1}{2}\frac{i(g^2+g'^2)\Lambda_W^2}{(4\pi)^2} \times \sum_\psi [(K_{mm}-L_{mm}) + P_{mm}(2c_w^4 + s_w^4 32(Q-T^3)^2 - 16 s_w^2 c_w^2 T^3(Q-T^3))].$$

- The pure photon sector gives

$$-i\Pi^T_{Af} = -\frac{1}{2}\frac{i(g^2+g'^2)\Lambda_W^2}{(4\pi)^2} c_w^2 s_w^2 \times \sum_\psi P_{mm}(2 + 32(Q-T^3)^2 + 16T^3(Q-T^3)).$$

- We observe that Π^T_A (0)= 0, guaranteeing a massless photon.
- Finally we obtain for the mixing sector:

$$-i\Pi^T_{AZf} = -\frac{1}{2}\frac{i(g^2+g'^2)\Lambda_W^2}{(4\pi)^2}c_w^2 s_w^2 \times \sum_\psi P_{mm}[2c_w^2 - 32s_w^2(Q-T^3)^2 - 16T^3(Q-T^3)(s_w^2-c_w^2)].$$

- We make the identification

$$m_V^2 = \Pi^T_f.$$

- This allows us to calculate the masses of the $W^\pm$ and Z^0 bosons or, conversely, use their experimentally known masses to calculate Λ_W.

5. Calculation Of The ρ Parameter And Λ_W

• When we consider the scattering of longitudinally polarized vector bosons, the vector boson propagator reads

$$iD^{\mu\nu}(p^2) = \frac{-i\eta^{\mu\nu}}{p^2 - \Pi_f^T(p^2)},$$

This differs from the vector boson propagator of the standard model in that the squared mass m^2_V of the vector boson is replaced by the self-energy term Π^T_f. For an on-shell vector boson, demanding agreement with the standard model requires that the following consistency equation be satisfied:

$$m_V^2 = \Pi_f^T(m_V^2).$$

$$-i\Pi_{Zf}^T = -\frac{1}{2}\frac{i(g^2 + g'^2)\Lambda_W^2}{(4\pi)^2} \times \sum_\psi [(K_{mm} - L_{mm}) + P_{mm}(2c_w^4 + s_w^4 32(Q - T^3)^2 - 16 s_w^2 c_w^2 T^3(Q - T^3))].$$

• It contains terms that include the electroweak coupling constant, the Weinberg angle, fermion masses, and the Λ_W parameter. As all these except Λ_W are known from experiment, the equation

$$m_Z^2 = \Pi^T_{Zf}(m_Z^2),$$

the right-hand side of which contains Λ_W can be used to determine $\Lambda_{W.}$ Using

$$g = 0.649, \qquad \sin^2\theta_w = 0.2312, \qquad m_t = 171.2 \text{ GeV}, \; m_Z = 91.1876 \pm 0.0021$$

we get

$$\Lambda_W = 541.9 \text{ GeV},$$

• Knowing Λ_W allows us to solve the consistency equation for the W-boson mass. Treating m_W as unknown, we solve using

$$-i\Pi^T_{W^\pm f} = -\frac{ig^2\Lambda_W^2}{(4\pi)^2}\sum_{q^L}(K_{m_1m_2} - L_{m_1m_2} + 2P_{m_1m_2}).$$

$$m_W \simeq 80.05 \text{ GeV}.$$

• This result, which does not incorporate radiative corrections $\Delta\rho$, is actually slightly closer to the experimental value m_W = 80.398 ± 0.025 GeV than the comparable tree-level standard model prediction m_W = 79.95 GeV, obtained using ρ = 1 where

$$\rho = \frac{m_W^2}{m_Z^2 \cos^2\theta_w},$$

We get from our model

$$\rho \simeq 1.0023.$$

which agrees well with estimates from the experimental ratio of neutral to charged currents.

• It is anticipated that our result for m_W (correct to 0.5%) will reach the correct value when radiative corrections $\Delta\rho$(RC) are included, for our regularization scheme will introduce some suppression of higher-order corrections at the energy scale of m_W.

For the boson masses, we have (PDG, Phys. Lett. B667, 1 (2008)):

m_W = 80.398 +/- 0.023,
m_Z = 91.1876 +/- 0.0021.

For $sw^2=\sin^2$ theta $_w$, we have the following values:

sw^2 = 0.23120 +/- 0.00015 (MS-bar scheme),

sw^2 = 0.2397 +/- 0.0013 (Møller scattering),

sw^2 = 0.2231 (on shell).

For the last of these values, there are no error bars. The corresponding values of ρ are

ρ(MS)= 1.0111 +/- 0.0008 (13.5 σ from 1.0),

ρ(Møller) = 1.0224 +/- 0.0024 (9.4 σ from 1.0),

ρ(onshell) = 1.0006 +/- 0.0006 (0. 9 σ from 1.0).

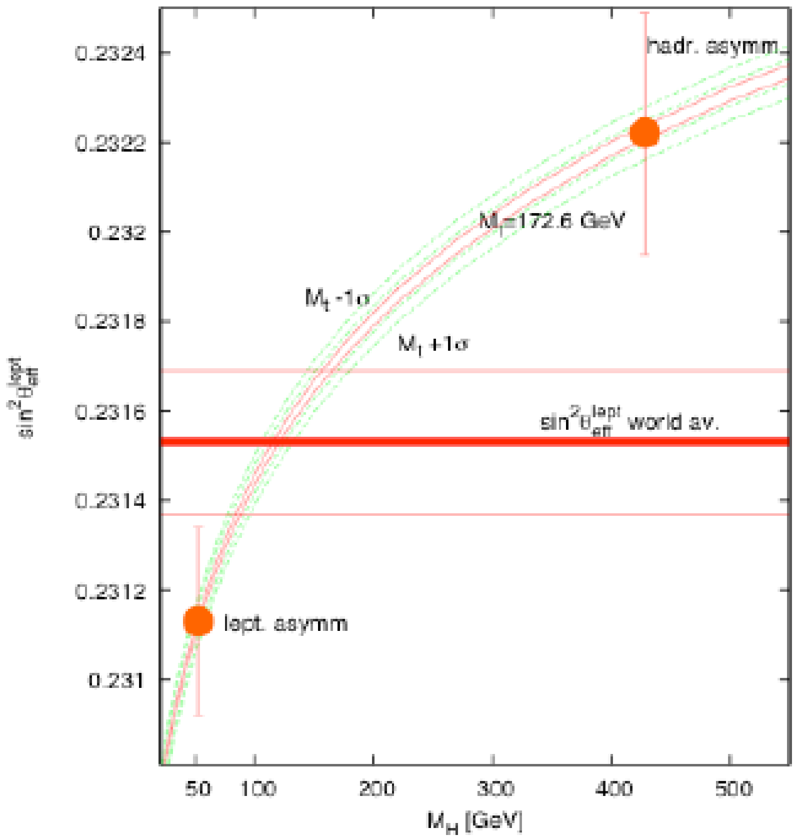
G. Altarelli

Plot $\sin^2\theta_{eff}$ vs m_H

P. Gambino

Exp. values are plotted at the m_H point that better fits given m_{texp}

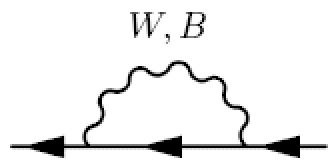
$\sin^2\theta_{eff}$



Clearly leptonic and hadronic asymm.s push m_H towards different values

6. Fermion Masses

• We will generate fermion masses from the finite one-loop fermion self-energy graph:



• This method of deriving fermion masses is more economical in assumptions, as we obtain the masses from our original massless electroweak Lagrangian by calculating fermion self-energy graphs (JWM, ArXiv 0709.4269 [hep-ph] (2007); JWM and V. T. Toth, ArXiv 0812.1991 [hep-ph] (2008).)

• A fermion particle obeys the equation:

$$\not{p} - m_{0f} + \Sigma(p) = 0, \qquad \not{p} - m_f = 0.$$

Here, m_{0f} is the bare fermion mass, m_f is the observed fermion mass and $\Sigma(p)$ is the finite proper self-energy part. We have

$$m_f - m_{0f} = \Sigma(p, m_f, g, \Lambda_f)|_{\not{p}-m_f=0},$$

where Λ_f denotes the energy scales for lepton and quark masses.

- A solution can be found by successive approximations starting from the bare mass m_{0f} , but we can also find a solution for $m_f \neq 0$ when $m_{0f} = 0$ for a broken symmetry vacuum state.

- The one-loop correction to the self-energy of a fermion with mass m_f in the regularized theory for a massive vector field is obtained from $\Sigma(p)$.

• We now identify the fermion mass as $m_f = \Sigma(0)$:

$$m_f = \frac{g^2}{4\pi^2} \exp\left(\frac{-m_V^2}{\Lambda_f^2}\right) m_f \left[E_1\left(\frac{2m_f^2}{\Lambda_f^2}\right) - \frac{m_V^2}{\Lambda_f^2}\int_2^\infty d\tau \exp\left(\tau\frac{m_V^2 - m_f^2}{\Lambda_f^2}\right) E_1\left(\tau\frac{m_V^2}{\Lambda_f^2}\right)\right].$$

• In addition to admitting a trivial solution at $m_f = 0$, this equation also has non-trivial solutions that can be computed numerically. We work with a single massless vector boson.

• A solution is obtained when

$$m_f = \frac{g^2}{4\pi^2} m_f E_1\left(\frac{2m_f^2}{\Lambda_f^2}\right).$$

$$\frac{m_f}{\Lambda_f} = \sqrt{\frac{1}{2}E_1^{-1}\left(\frac{4\pi^2}{g^2}\right)}.$$

• Using the electroweak coupling constant g ~0.649, we obtain for leptons

$$\Lambda_f \simeq 4.3 \times 10^{20} m_f.$$

• For quarks, we use the strong coupling constant g_s ~ 1.5, and also introduce a color factor 3. Thereafter, we obtain

$$\Lambda_f \simeq 35 m_f.$$

• For a top quark mass m_t = 171.2 GeV, the corresponding energy scale is about $\Lambda_t \sim 6$ TeV.

• In these calculations, Λ_f plays a role that is similar to that of the diagonalized fermion mass matrix in the standard model. The number of undetermined parameters, therefore, is the same as in the standard model: for each fermion a corresponding Λ_f determines its mass.

• In the standard EW Higgs model the masses of matter particles are obtained from a classical Yukawa interaction $g_i\, \varphi\psi\psi$ with a free coupling constant parameter for each quark and lepton. The classical Yukawa Lagrangian cannot determine the fermion mass spectrum of the standard model.

• In addition to fermion self-energy graphs, another case must be considered. Emission or absorption of a charged vector boson $W^{\pm}$ can be flavor violating, through the off-diagonal components of the CKM matrix.

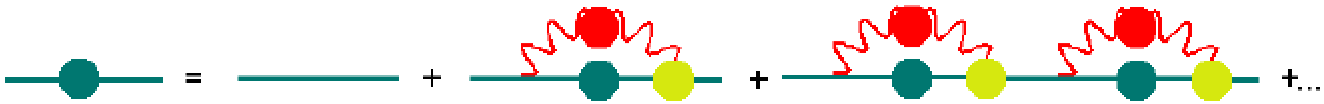
• In the standard model, such flavor violating terms are not considered significant, due to the smallness of the corresponding CKM matrix elements. However, in our regularized theory, additional factors ff′ enter into the picture in a manner similar to the self-energy calculation we just described. These may include terms that correspond to the off-diagonal elements of the neutrino mass matrix, offering a natural explanation for neutrino oscillations without having to introduce new interactions.

7. Schwinger-Dyson Equations And Fermion Masses

• To solve the fermion mass problem, we must consider the integral equation obtained from the Schwinger-Dyson equation (J. S. Schwinger, Proc. Nat. Acad. Sc. 37, 452 (1951); F. J. Dyson, Phys. Rev. 75, 1736 (1949); A. Raya, ArXiv: 0902.1791 [hep-ph])).

$$S_F^{-1}(p) = S_F^{(0)-1}(p) - ig^2 \int \frac{d^4k}{(2\pi)^4} \gamma^\mu S_F(k) \Gamma^\nu(k,p) D_{\mu\nu}(k-p)$$

where g^2 is the coupling, $D_{\mu\nu}$ represents the complete vector particle propagator and Γ^ν the full fermion-vector particle vertex.



• We can derive the Euclidean mass gap equation

$$m_{0f} = \frac{\alpha}{\pi^2} \int \frac{d^3k}{(k-p)^2} \frac{M(k,\Lambda_f)}{k^2 + M^2(k,\Lambda_f)} \qquad \alpha = g^2/4\pi$$

8. The Running Of Coupling Constants And Unitarity

• The Higgs field resolves the issue of unitarity, by precisely canceling out badly behaved terms in the tree-level amplitude of processes involving longitudinally polarized vector bosons, for instance $W^+_L W^-_L \rightarrow W^+_L W^-_L$ or $e^+e^- \rightarrow W^+_L W^-_L$. The challenge to any theory that aims to compete with the SM without introducing a Higgs particle is to generate the correct fermion and boson masses on the one hand, and ensure unitary behavior for these types of scattering processes on the other (JWM and V. T. Toth, ArXiv 0812.1994 [hep-ph]).

• Given the way Π^T appears in the vector boson propagator, it is reasonable to make the identification:

$$\Pi^T_{W\,f}(q^2) = m^2_W(q^2), \qquad \Pi^T_{Z\,f}(q^2) = m^2_Z(q^2).$$

• When we rewrite the theory's Lagrangian in terms of massive vector bosons, the Lagrangian picks up a finite mass contribution from the total sum of polarization graphs:

$$
\begin{aligned}
L_m &= \frac{1}{8}v^2g^2[(W_\mu^1)^2 + (W_\mu^2)^2] \\
&\quad + \frac{1}{8}v^2[g^2(W_\mu^3)^2 - 2gg'W_\mu^3B^\mu + g'^2B_\mu^2] \\
&= \frac{1}{4}g^2v^2W_\mu^+W^{-\mu} \\
&\quad + \frac{1}{8}v^2(W_{3\mu}, B_\mu)\begin{pmatrix} g^2 & -gg' \\ -gg' & g'^2 \end{pmatrix}\begin{pmatrix} W^{3\mu} \\ B^\mu \end{pmatrix}
\end{aligned}
$$

$$
m_W = \frac{1}{2}vg, \quad m_Z = \frac{1}{2}v(g^2 + g'^2)^{1/2}, \quad m_A = 0.
$$

• v is the electroweak symmetry breaking scale (which, in the SM, is the vacuum expectation value of the Higgs scalar).

• Consistency requires the running of the constants g and g'. Starting with the W mass, we obtain

$$
\frac{g^2(q^2)}{g^2(m_Z^2)} = \frac{\Pi_{Wf}^T(q^2)}{\Pi_{Wf}^T(m_Z^2)}. \qquad v^2 = \frac{4\Pi_{Wf}^T(m_Z^2)}{g^2(m_Z^2)} \simeq (245\ \mathrm{GeV})^2.
$$

Using the Z mass we obtain

$$\frac{g^2(q^2)+g'^2(q^2)}{g^2(m_Z^2)+g'^2(m_Z^2)}=\frac{\Pi^T_{Zf}(q^2)}{\Pi^T_{Zf}(m_Z^2)},$$

which establishes the running of $g'(q^2)$.

• These relationships also allow us to calculate the running of the Weinberg angle θ_w, which is defined through the ratio of the coupling constants g and g' as

$$\cos\theta_w=\frac{\sqrt{g^2+g'^2}}{g}.$$

$$\begin{aligned}\Pi^T_{Zf}(q^2) =& \frac{1}{2}\frac{(g_0^2+g_0'^2)\Lambda_W^2}{(4\pi)^2} \\ &\times\sum_{\psi}\{[K_{mm}(q^2)-L_{mm}(q^2)] \\ &\quad + P_{mm}(q^2)[2\cos^4\theta_w+32\sin^4\theta_w(Q-T^3)^2 \\ &\qquad - 16\sin^2\theta_w\cos^2\theta_w T^3(Q-T^3)]\},\end{aligned}$$

$$\Pi^T_{Wf}(q^2) = \frac{g_0^2\Lambda_W^2}{(4\pi)^2} \times \sum_{q^L}(K_{m_1m_2}(q^2) - L_{m_1m_2}(q^2) + 2P_{m_1m_2}(q^2)).$$

$$K_{m_1m_2}(q^2) = \int_0^{\frac{1}{2}} d\tau(1-\tau)\Bigg[\exp\left(-\tau\frac{q^2}{\Lambda_W^2} - f_{m_1m_2}\right) + \exp\left(-\tau\frac{q^2}{\Lambda_W^2} - f_{m_2m_1}\right)\Bigg],$$

$$P_{m_1m_2}(q^2) = -\frac{q^2}{\Lambda_W^2}\int_0^{\frac{1}{2}} d\tau\tau(1-\tau)\Bigg[E_1\left(\tau\frac{q^2}{\Lambda_W^2} + f_{m_1m_2}\right) + E_1\left(\tau\frac{q^2}{\Lambda_W^2} + f_{m_2m_1}\right)\Bigg],$$

$$L_{m_1m_2}(q^2) = \int_0^{\frac{1}{2}} d\tau(1-\tau)\Bigg[f_{m_1m_2}E_1\left(\tau\frac{q^2}{\Lambda_W^2} + f_{m_1m_2}\right) + f_{m_2m_1}E_1\left(\tau\frac{q^2}{\Lambda_W^2} + f_{m_2m_1}\right)\Bigg],$$

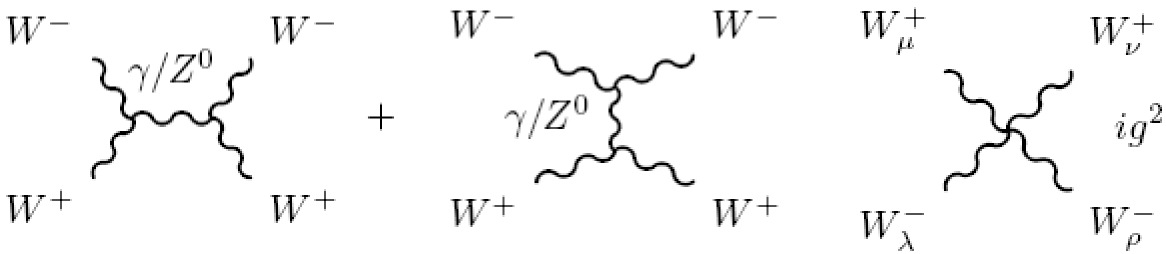
$$f_{m_1m_2} = \frac{m_1^2}{\Lambda_W^2} + \frac{\tau}{1-\tau}\frac{m_2^2}{\Lambda_W^2}.$$

• The mass and width of the Z boson, m_Z and Γ_Z, and its couplings to fermions, for example the ρ parameter and the effective electroweak mixing angle for leptons,
are precisely measured (http://www.cern.ch/LEPEWWG):

$$
\begin{aligned}
m_Z &= 91.1875 \pm 0.0021 \text{ GeV} \\
\Gamma_Z &= 2.4952 \pm 0.0023 \text{ GeV} \\
\rho_\ell &= 1.0050 \pm 0.0010 \\
\sin^2\theta_{\rm eff}^{\rm lept} &= 0.23153 \pm 0.00016\,.
\end{aligned}
$$

• Higgless model calculation: $\rho = 1.0023 + \Delta\rho(RC)$

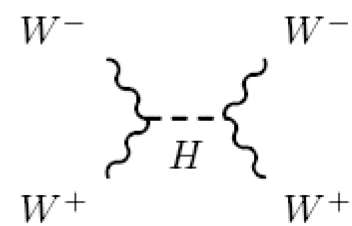
• The scattering of two longitudinally polarized W vector bosons can take place through one of the following processes:



$$ig^2(2\eta_{\mu\nu}\eta_{\lambda\rho} - \eta_{\mu\lambda}\eta_{\nu\rho} - \eta_{\mu\rho}\eta_{\nu\lambda}).$$

• In the high energy limit, the SM with a Higgs yields the matrix element

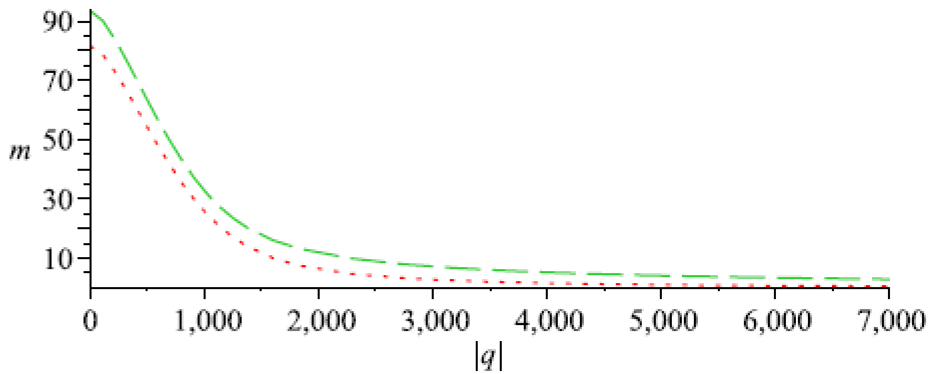
$$i\mathcal{M}_{\rm SM}(W_L^+W_L^- \to W_L^+W_L^-) =$$
$$ig^2\left[\frac{\cos^2\theta + 3}{4\cos\theta_w^2(1-\cos\theta)} - \frac{m_H^2}{2m_W^2} + \mathcal{O}(s^{-1})\right].$$



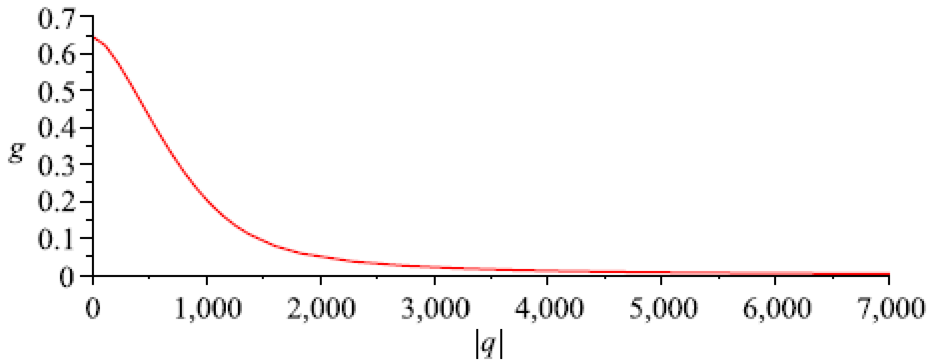
• In the Higgless model we get

$$i\mathcal{M}_M = i\mathcal{M}_s + i\mathcal{M}_t + i\mathcal{M}_4 \qquad (38)$$
$$= ig^2\left[\frac{(\cos\theta + 1)(4m_W^2 - 3\Pi_{Zf}^T\cos^2\theta_w)}{8m_W^4}s + \mathcal{O}(1)\right].$$

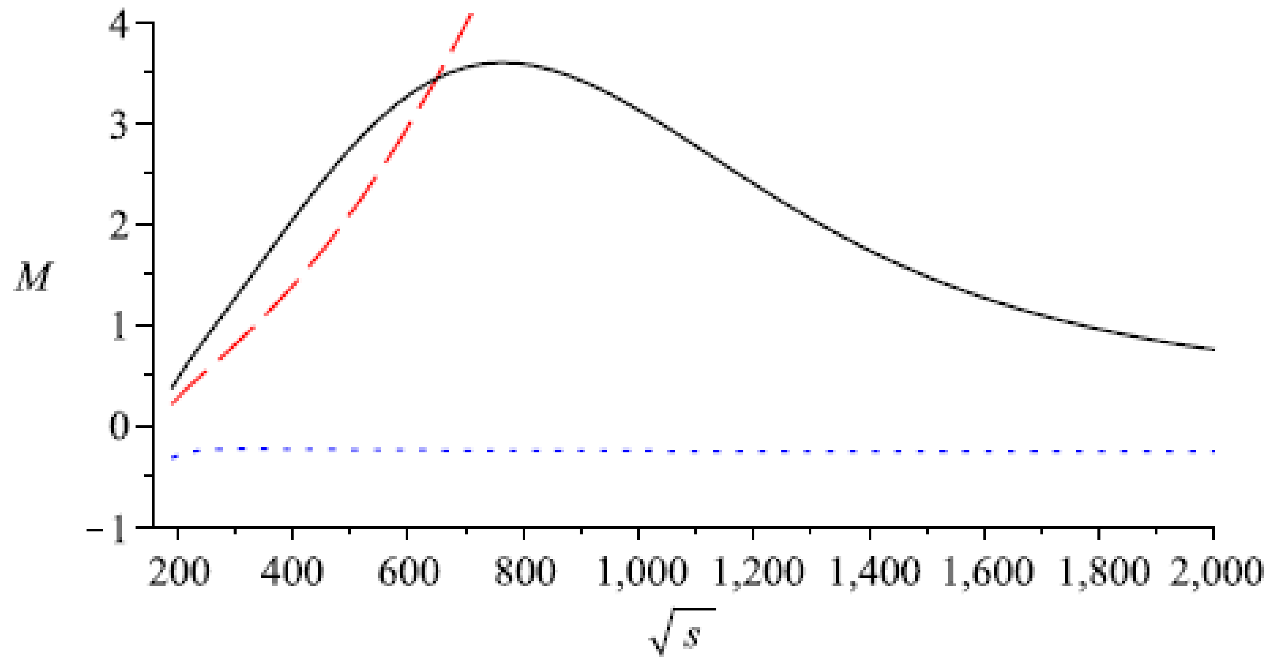
• In the case of the Higgless FEW theory, no additive cancellation takes place. However, the running of the electroweak coupling constant is such that at high s, g(s)s ~ const., which is sufficient to ensure that unitarity is not violated.



The running of the *W mass (red dotted line) and Z* mass (dashed green line) as functions of momentum. Both axes are measured in units of GeV.

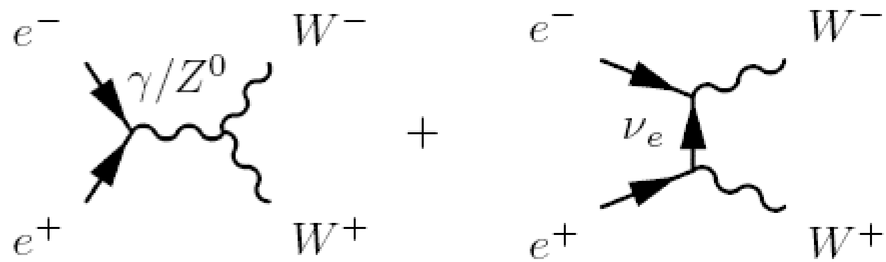


The running of the electroweak coupling constant *g* as a function of momentum, measured in GeV.



The tree-level scattering amplitude of longitudinal *$W^{\pm}$ bosons at a scattering angle $\theta = \pi/2$. The SM (blue* dotted line) predicts an asymptotically constant amplitude at high energy. Without the Higgs particle (red dashed line) the amplitude is divergent. In the FEW theory (black solid line) this divergent amplitude is suppressed by the running of the electroweak coupling constant.

• The production of W^+W^- pairs from electron-positron collisions can take place via one of the following processes:

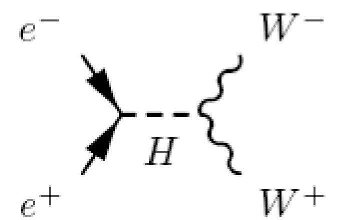


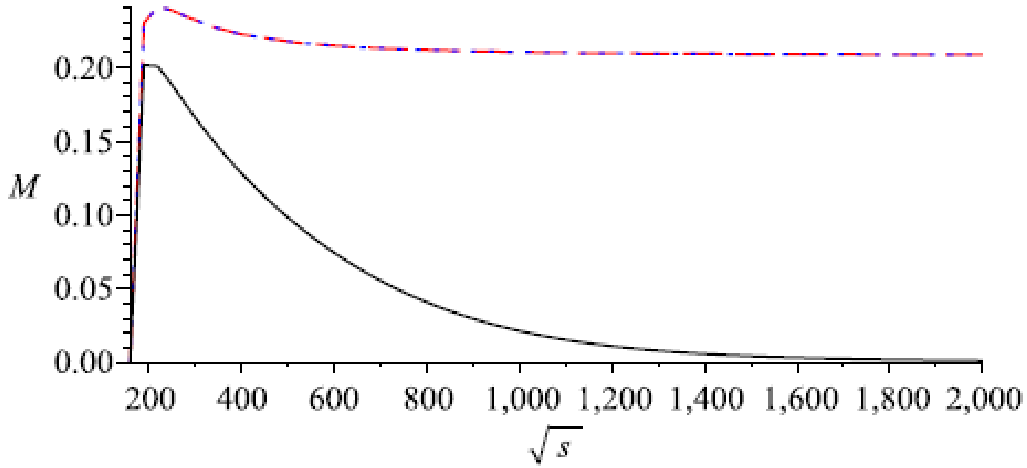
• In the high-energy limit, we get in the Higgless model:

$$i(\mathcal{M}_s + \mathcal{M}_t) = -ig^2 \left[\frac{m_e}{2m_W^2} \sqrt{s} + \mathcal{O}(1) \right].$$

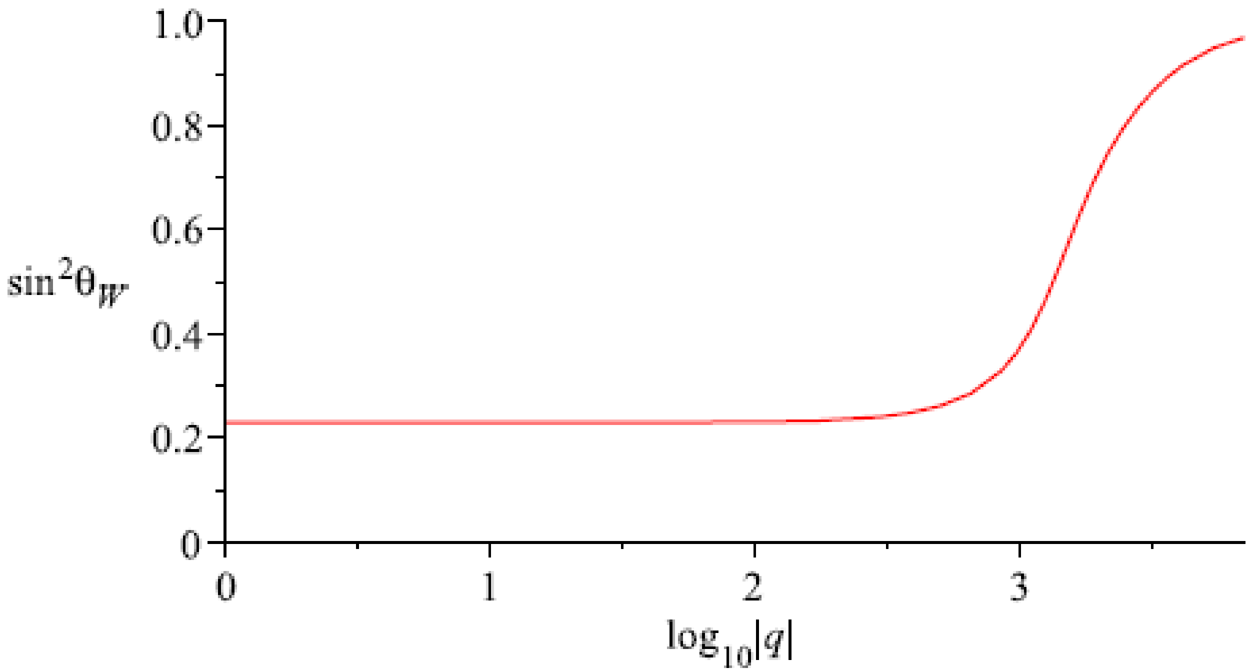
• In the SM with a Higgs:

$$i\mathcal{M}_H = ig^2 \left[\frac{m_e}{2m_W^2} \sqrt{s} + \mathcal{O}(1) \right]$$





The scattering amplitude, at a scattering angle of
$\theta = \pi/2$, of electrons and positrons annihilating into longi-
tudinally polarized *$W^{\pm}$ bosons as a function of the center-*
of-mass energy *$\sqrt{s}$, measured in GeV. The SM result (blue*
dotted line) is indistinguishable from the SM result that was
calculated without the Higgs particle (red dashed line), as due
to the smallness of *m_e, the divergent term that is proportional*
to *$m_e\sqrt{s}$ does not begin to dominate until much higher en-*
ergies. Our Higgless theory (black line) predicts a significant
suppression of the amplitude even at moderate energies.



The running of the Weinberg angle as a function of momentum in the Higgless model, measured in GeV.

9. Conclusions

• An electroweak model without a Higgs particle that breaks $SU_L(2) \times U_Y(1) \rightarrow U(1)_{em}$ has been developed, based on a finite quantum field theory. We begin with a massless and gauge invariant theory that is UV complete, Poincaré invariant and unitary to all orders of perturbation theory. A fundamental energy scale Λ_W enters into the calculations of the finite Feynman loop diagrams. A path integral is formulated that generates all the Feynman diagrams in the theory. The self-energy boson loop graphs with internal fermions comprised of the observed 12 quarks and leptons
have an associated quantum measure in the path integral that is broken to generate 3 Nambu-Goldstone modes that give the $W^{\pm}$ and the Z^0 bosons masses, while retaining a zero mass photon.

• There is no classical Higgs scalar field particle and no new particles are included in the particle spectrum. All particle masses are generated by QFT self-energy diagrams.

• The $W_LW_L \rightarrow W_LW_L$ and $e^+e^- \rightarrow W^+_L W^-_L$ amplitudes do not violate unitarity at the tree graph level due to the running with energy of the electroweak coupling constants g, g′ and e.
This is essential for the physical consistency of the model as is the case in the standard Higgs electroweak model.

• A self-consistent calculation of the energy scale yields $\Lambda_W = 542$ GeV and a prediction of the W mass from the W-boson self-energy diagrams in the symmetry broken phase gives $m_W = 80.05$ GeV, which is accurate to 0.5%. Radiative corrections $\Delta\rho(RC)$ remain to be calculated.

• The EW cosmological constant problem is solved without fine-tuning.

• The Higgs mass hierarchy problem is solved without fine-tuning.

• The origin of mass in the universe is due to self-consistent solutions of QFT self-energies – not to a classical scalar Higgs field and Yukawa interactions.

END